values.

The mass transfer between fluid elements is considered to be caused mainly by coalescence and dispersion, and thus the overall mass transfer coefficient may be dependent on frequency of coalescence and dispersion, contact time, and contact area of fluid elements. Although U might be expected to vary with agitation speed, we assumed U to be a constant. The values of $\overline{Y}$ were computed for parametric values of U, with $k = 1.10$, $B = 0.10$, $\kappa = 0.10$, $I_1 = 297$, and $I_2 = 1.0$. In Figure 4, the calculated values of $\overline{Y}$ are plotted vs. the period T together with the experimental results by using $f = 0.5$ for baffled and 0.25 for unbaffled vessels. A comprehensive geometrical model has not been made. For periods greater than 10 sec., the calculated reaction rates are relatively insensitive to period but are dependent upon the mass transfer rate. At small periods, the reaction rates is insensitive to the mass transfer rates.

The analytical and experimental methods for the chloral hydrate system indicate sufficient concurrence to merit reporting the simple approaches used for the model.

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NOTATION

A = defined by Equation (3), mole/(liter (min.) or mho./(cm.) (min.)
a = defined by Equation (9)
B = defined by Equation (3), min./rad.
b, c = defined by Equation (9)
C = product
D = diameter of agitator, cm.
d = defined by Equation (9)
E = reactant
f = coefficient of impeller discharge.
G = G value of initiation reaction, mole/rad.
I = dose rate, rad./min.
I_1, I_2 = dose rate of high and low dose rate portion, rad./min.
$\overline{I}$ = average dose rate in the reactor, rad./min.
i = number of cycle times per fluid element in the reactor
K = apparent rate constant of the propagation reaction, sec.$^{-1}$
k = apparent rate constant of the termination reaction, sec.$^{-1}$
k_1, k_2 = propagation and termination rate constant, liter/(mole) (sec.)
l = termination rate constant, liter/(mole) (sec.)
M = reactant
N = agitation speed, rev./sec.
n = concentration of chain carrier, mole/liter; $[R\cdot]$ = concentration of $R\cdot$ designated by n
$\overline{n}$ = defined by Equation (6), mole/liter
p = defined by Equation (9)
Q = discharge rate of agitator, cc./sec.
$R\cdot$ = chain carrier radical
r, s = solutions of Equations (11) and (12)
S = scavenger
T = period of motion of fluid element in the reactor, sec.
t = time, sec.
U = overall mass transfer coefficient based on interaction between fluid elements, sec.$^{-1}$
V = reactant volume, cc.
X = rate of production of the product, mole/(liter) (min.) or mho./(cm.) (min.)
Y = X/A, dimensionless reaction rate
$\overline{Y}$ = defined by Equation (9).

Greek Letters

κ = fraction of high dose rate portion in the reactor
θ = mean circulation time of fluid in the reactor, sec.

Subscripts

n = nonmixing
p = perfect mixing

LITERATURE CITED

1. Hill, F. B., and R. M. Felder, *AIChE J.*, **11**, 873 (1965).
2. Kawakami, W., K. Yoshida, and T. Sugo, preprint of the annual meeting of Soc. Chem. Engrs., Japan, p. 297 (1965).
3. Fredrickson, A. G., A. H. Brown, R. C. Miller, and H. M. Tsuchiya, *Am. Rocket Soc. J.*, **31**, 1429 (1961).
4. Hill, F. B., and N. Reiss, *Can. J. Chem. Eng.*, **46**, 124 (1968).
5. ———, and L. H. Shendalman, *AIChE J.*, **14**, 798 (1968).
6. Yemin, L., and F. B. Hill, *Ind. Eng. Chem. Fundamentals*, **8**, 210 (1969).
7. Andrews, H. C., and P. A. Shore, *J. Chem. Phys.*, **18**, 1165 (1950).
8. Freeman, G. R., A. B. Van Cleave, and J. W. T. Spinks, *Can. J. Chem.*, **31**, 1164 (1953).
9. Lotz, A. P., and K. Schmidt, *Nature*, **201**, 493 (1964).
10. McIntosh, R. G., R. C. Eager, and J. W. T. Spinks, *Can. J. Chem.*, **42**, 2033 (1964).
11. Kawakami, W., and H. S. Isbin, *ibid.*, **47**, 4531 (1969).
12. Hill, F. B., P. T. Fallon, A. C. Muller, and M. A. Rothbart, *IAEA/SM-123/44* (1969).
13. Kamiwano, M., K. Yamamoto, and S. Nagata, *Chem. Eng. Japan*, **31**, 365 (1967).
14. Corrsin, Stanley, *AIChE J.*, **10**, 870 (1964).
15. Mohr, W. D., "Proceeding of Thermoplastic Materials," E. C. Bernhardt, ed., p. 117, Reinhold, New York (1960).
16. Rushton, J. H., D. E. Mack, and H. J. Everett, *Trans. Am. Inst. Chem. Engrs.*, **42**, 441 (1946).
17. Sacks, J. P., and J. H. Rushton, *Chem. Engr. Progr.* **50**, 597 (1954).
18. Nagata, S., K. Yamamoto, and M. Ujihara, *Mem. Fac. Eng. Kyoto Univ.*, **20**, 336 (1958).
19. *Ibid.*, **21**, 260 (1959).
20. Marr, G. R., and E. F. Johnson, *AIChE J.*, **9**, 383 (1963).

Drag Reduction Correlations

G. K. PATTERSON, J. L. ZAKIN, and J. M. RODRIGUEZ
University of Missouri, Rolla, Missouri

We would like to point out a more general drag reduction correlation to which the one presented by Astarita, Greco, and Nicodemo (*1*) is very similar. The older correlation, which was proposed and tested for many polymers and solvents by Rodriguez, Zakin, and Patterson (*2*, *3*) was of the form f/f_{pv} vs. $U\tau_1/D^{0.2}$, where τ_1 is the first-mode Zimm relaxation time of the polymer molecule in solution. The *A, G, N* correlation was f/f_{pv} vs. $\nu/\nu_{0.5}$, where $\nu = UN_{Re}^{0.75}/D$, and $\nu_{0.5}$ is the value of ν at $f/f_{pv} = 0.5$. Thus, all drag reduction data were forced to fit the correlation at $f/f_{pv} = 0.5$, requiring a set of drag reduction data for each polymer solution to establish the value of $\nu_{0.5}$. The *R, Z, P* correlation established that for polymers dissolved in polar or nonpolar solvents, the $U\tau_1/D^{0.2}$ abscissa accounts for concentration and diameter effects, even though the levels of $U\tau_1/D^{0.2}$ vary among polymer-solvent systems.

Astarita, Greco, and Nicodemo obtained a linear log-log relationship between $1/\nu_{0.5}$, relative measure of solution relaxation time, and concentration up to a limiting value for their ET 597 solutions. They found that for dilute solutions, $1/\nu_{0.5} \propto c^{2.5}$. It is not known whether this relation is applicable to other polymer solutions. The concentration effect is accounted for in the *R, Z, P* correlation by its effect on the calculated Zimm relaxation time τ_1. All dilute polymer solutions tested so far are correlated by that relaxation time relationship, which requires viscosity-concentration data for calculation.

The diameter effect on drag reduction is accounted for in each of the correlations by a velocity-to-diameter (to some power) ratio. In the *A, G, N* correlation, the abscissa contains the ratio $U^{1.75}/D^{0.25}$; the *R, Z, P* correlation contains $U/D^{0.2}$. Since the relative powers of U and D are the important factors, the *A, G, N* and *R, Z, P* correlations may be compared as U^7/D and U^5/D, respectively. It is doubtful that this difference can be seen in drag reduction data over the velocity ranges normally encountered.

The form of the *R, Z, P* correlation is supported by considerations of the turbulent energy dissipation spectrum (*4*). For a Maxwell fluid and isotropic turbulence with turbulence intensity equal to that of a purely viscous fluid, the ratio of energy dissipation rates was found to be

$$\frac{W}{W_s} = \frac{\mu \int_0^\infty [n^2F(n)/(1 + 4\pi^2n^2\beta^2u'^2\mathrm{T}^2/u^2)]dn}{\mu_s \int_0^\infty n^2F(n)dn} \qquad (1)$$

Some phenomenological considerations led to the following relationship between wall shear stress and turbulent energy dissipation rate:

$$\tau_w/(\tau_w)_s = (W/W_s)^{0.8}(\mu_s/\mu)^{0.6} \qquad (2)$$

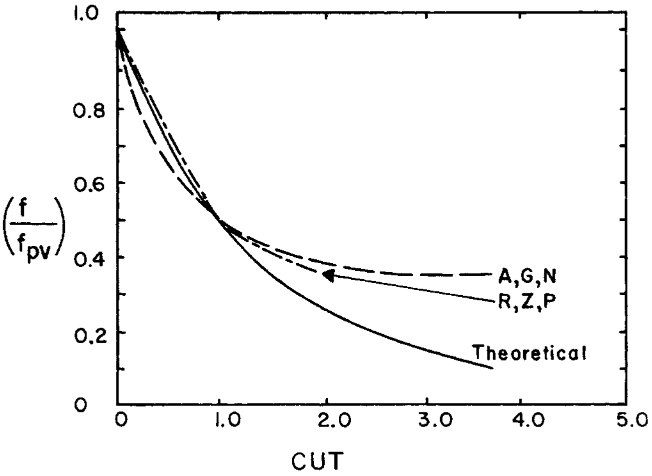

Fig. 1. Comparison of *R, Z, P* and *A, G, N* correlations with theoretical result.

The friction factor ratio, which is the friction factor of the viscoelastic fluid divided by the friction factor of a purely viscous fluid of the same viscosity, may be expressed as

$$\frac{f}{f_{pv}} = \frac{\tau_w}{(\tau_w)_s}\left(\frac{\nu_s}{\nu}\right)^{0.2} = \left[\frac{\int_0^\infty \frac{n^2F(n)dn}{1 + n^2C^2T^2}}{\int_0^\infty n^2F(n)dn}\right]^{0.8} \qquad (3)$$

A simple energy spectrum function is $F(n) \propto n^{-4}$ for $n = kU$ (independent of diameter). If n, the relative frequency, is scaled such that at $n < 1$ the value of $F(n) \rightarrow 0$, the above integrals may be evaluated from 1 to ∞ to obtain

$$f/f_{pv} = [1 - (CUT)(n/2 - \tan^{-1}(CUT))]^{0.8} \qquad (4)$$

The shape of this function is shown in Figure 1 and is compared with the shape of the correlation (which has been forced to have $f/f_{pv} = 0.5$ at $CUT = 1.0$) by choosing the proper value of C. The *A, G, N* correlation is plotted with $CUT = \nu/\nu_{0.5}$.

The most obvious deficiency of the predicted f/f_{pv} function is that it is asymptotic to zero, whereas the correlation seems asymptotic to $f/f_{pv} = 0.2$ to 0.4, depending on the polymer-solvent system (*1, 3*).

NOTATION

c = concentration
C = constant replacing $4\pi^2u'^2\beta^2/\bar{u}^2$
D = pipe diameter
f = friction factor
f_{pv} = friction for purely viscous fluid
$F(n)$ = Eulerian energy spectrum function
M_v = viscosity average molecular weight
n = velocity fluctuation in Eulerian energy spectrum
N_{Re} = generalized Reynolds number
R = thermodynamic constant
T = Maxwell fluid relaxation time
t = temperature
$\bar{u}$ = time average velocity
u' = root-mean-square fluctuating velocity
U = bulk mean velocity
W = turbulent energy dissipation rate for solution
W_s = turbulent energy dissipation rate for solvent

Greek Letters

β = constant in Equation (1)
λ_k = eigenvalue in Zimm equation
η_{sp} = specific viscosity, $(\mu/\mu_s) - 1$
μ = solution viscosity
μ_s = solvent viscosity
ν = eddy frequency used in *A, G, N* correlation
$\nu_{0.5}$ = eddy frequency at $f/f_{pv} = 0.5$
τ_1 = Zimm relaxation time, $\dfrac{M_v(\eta_{sp}/c)\mu_s}{0.586\,Rt\lambda_k}$
τ_w = pipe wall shear stress for solution
$(\tau_w)_s$ = pipe wall shear stress for solvent

LITERATURE CITED

1. Astarita, Gianni, Guido Greco, Jr., and Luigi Nicodemo, *AIChE J.*, **15**, 564 (1969).
2. Rodriguez, J. M., M. S. thesis, Univ. Mo., Rolla (1966).
3. ———, J. L. Zakin, and G. K. Patterson, *Soc. Petrol. Engrs. J.*, **7**, 325 (1967).
4. Patterson, G. K., and J. L. Zakin, *AIChE J.*, **14**, 434 (1968).